Reply

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We thank N. Kuzmina for her interesting comments (Kuzmina 2000) on "Effects of Baroclinicity on Double-Diffusive Interleaving" (May and Kelley 1997, hereafter MK97). The comments have stimulated additional thought (and work) on our part. We hope that readers will be interested in the issues that have been raised.

Kuzmina's first comment regards our discussion (in MK97) of an earlier paper by Kuzmina and Rodionov (1992, hereafter KR92). In MK97, we stated that vertical advection of the background velocity field (i.e., $w'\overline{v}_z$) was neglected by KR92. A careful reading of KR92 reveals that the term appeared in the equations of motion (19), was neglected in the derivation of (26) and plotting of Fig. 2, and was subsequently reintroduced. We should have been more specific in our statement regarding the exclusion of this term.

Kuzmina's second comment regards a form of instability discussed by MK97 (section 4d). Though the instability has baroclinicity as its fundamental energy source, it differs significantly from instabilities predicted by either McIntyre (1970) or KR92. Unlike the McIntyre case, the instability criterion does not depend on the frontal Richardson number or the Prandtl number. Unlike the KR92 case, the instability occurs with isopycnal and isohaline slope of opposite sign. Because of these significant differences, we consider our case to be a new form of instability.

Kuzmina's third comment regards the derivation of the low-shear limit in MK97 (section 3f). Referring to the conservation equation for density,

$$\frac{\partial\rho'}{\partial t} + \overline{v}\frac{\partial\rho'}{\partial y} + u'\overline{\rho}_x + w'\overline{\rho}_z - (1 - \gamma_f)\rho_o\beta K_s\frac{\partial^2 S'}{\partial z^2} = 0, \qquad (1)$$

she questioned the process used to eliminate advection of the perturbation density field by the background flow (i.e., $\overline{v}\partial\rho'/\partial y$, where $\overline{v} = v_o + \overline{v}_x x + \overline{v}_z z$), without dropping horizontal advection of the background density field by the perturbation flow (i.e., $u'\overline{\rho}_x$). Both terms are related to baroclinicity, the former through the vertical shear $\overline{v}_z$, and the latter through the horizontal density gradient $\overline{\rho}_x$.

First, we would like to point out that MK97 did not assume "$u'\overline{\rho}_x \sim \partial\rho'/\partial t$," as suggested by Kuzmina. In the low-shear limit, the dominant terms in the density conservation equation are time-dependence ($\partial\rho'/\partial t$), vertical advection ($w'\overline{\rho}_z$), and vertical mixing $[-(1 - \gamma_f)\rho_o\beta K_s\partial^2 S'/\partial z^2]$. In this limit, the terms arising from baroclinicity ($\overline{v}\partial\rho'/\partial y$ and $u'\overline{\rho}_x$) are both small in comparison to the dominant terms.

Second, in order to clarify the method used to eliminate terms arising from advection of the perturbation fields by the background flow (e.g., $\overline{v}\partial\rho'/\partial y$), we would like to elaborate on the derivation of MK97. We will show that the background advection terms drop out of the equations of motion, automatically, provided that the wavenumbers have the time-dependence given by MK97.

Following MK97, we introduce perturbations that are harmonic in space. However, now we include explicitly time-dependent wavenumbers, as follows:

$$\rho' = \hat{\rho}\exp i[kx + l(y - v_o t) + mz], \qquad (2)$$

where $k = k(t)$, $l = l(t)$, and $m = m(t)$ are the wavenumbers in the cross-front, along-front, and vertical directions, respectively. The along-front dependence [i.e., $(y - v_o t)$] takes into account translation by the mean background velocity. Growth of the intrusions is included in the coefficient $\hat{\rho} = \hat{\rho}(t)$ and corresponding quantities $\hat{u}(t)$, $\hat{v}(t)$, $\hat{w}(t)$, $\hat{p}(t)$, and $\hat{S}(t)$. Exponential growth is not assumed at this point.

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Substitution into the density conservation equation (1) yields

$$\frac{d\hat{\rho}}{dt} + i\left[\frac{dk}{dt}x + \frac{dl}{dt}(y - v_o t) - lv_o + \frac{dm}{dt}z\right]\hat{\rho} + [v_o + \overline{v}_x x + \overline{v}_z z]il\hat{\rho} + \hat{u}\overline{\rho}_x + \hat{w}\overline{\rho}_z + (1 - \gamma_f)\rho_o \beta K_s m^2 \hat{S} = 0. \quad (3)$$

Following MK97 (section 3d), we assume the wavenumbers adjust as the intrusive field is rotated by the background shear, according to: $dk/dt = -l\overline{v}_x$, $dl/dt = 0$, and $dm/dt = -l\overline{v}_z$. In this case, the terms proportional to x, $(y - v_o t)$, and z cancel, yielding

$$\frac{d\hat{\rho}}{dt} + \hat{u}\overline{\rho}_x + \hat{w}\overline{\rho}_z + (1 - \gamma_f)\rho_o \beta K_s m^2 \hat{S} = 0. \quad (4)$$

Comparison of (4) and (1) reveals that the term arising from advection of the perturbation density field by the background flow (i.e., $\overline{v}\partial\rho'/\partial y$) has been eliminated. Similarly, the corresponding terms drop out of the momentum and salinity conservation equations. This shows that advection of the perturbation fields by the background flow leads to a temporal tilting of the interleaving layers (i.e., prescribed by the time-dependent wavenumbers). Importantly, it has no other effect on the interleaving dynamics. In contrast, terms arising from advection of the background fields by the perturbation flow remain in the equations of motion. In particular, the term $u'\overline{\rho}_x$ remains in the conservation equation for density. There is, therefore, no need to neglect this term from the analysis, as suggested by Kuzmina.

It is worth noting that the terms arising from advection by the background flow can be eliminated, as shown above, regardless of the magnitude of the background shear. Having done so, the challenge, then, is to solve for the intrusion growth, given time-dependent wavenumbers in the equations of motion. In MK97, the high-shear and low-shear limits were introduced to address two cases in which the wavenumbers are roughly constant over the time scale of intrusion growth. In both limits, exponentially growing solutions [i.e., $\hat{\rho}(t) \propto \exp\lambda t$] could be considered. In intermediate cases, not discussed by MK97, the wavenumbers vary as the intrusions grow and the time dependence is much more complicated. This behavior is a current topic of our research.

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